A quasi-proportional electoral system "only for honest men"? The hidden potential for manipulating mixed compensatory electoral systems

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Abstract
Mixed compensatory systems have risen in popularity in recent years. Under such systems, single-seat districts elect only the leader of the local suffrage, but the system nevertheless produces a (nearly) proportional outcome overall, via compensatory mandates. Elections in Albania, Italy, Lesotho and Venezuela, however, demonstrate a particular loophole for such systems: strategic voting, organised by political parties. Large parties can achieve over-representation by encouraging their voters to split their votes. In this way, they outsmart the compensatory mechanism designed to lead to proportional results. These disproportional results are particularly controversial, since they are deliberate and strategic. This study takes the 2005 Albanian elections as its main case study, and uses simulations to illustrate its political consequences.

Keywords: Electoral systems; Albania; mixed compensatory systems; vote splitting; strategic voting.

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Introduction: The prototype of the 21st century’s electoral system?

Since 1989, the German electoral system has become increasingly popular among countries reforming their electoral systems. Eight countries have introduced mixed compensatory electoral systems for their national elections: Albania, Bolivia, Hungary, Lesotho, Mexico, New Zealand, Romania, Venezuela (Reynolds, Reilly, & Ellis, 2005: 91, updated by the author). Others discuss its introduction or apply the same system at lower levels of administration. Mixed systems are praised for combining "the best of both worlds" (Shugart & Wattenberg, 2001: 582-583). In several countries—specifically Albania, Italy, Lesotho, and Venezuela—an unexpected strategic behaviour has been observed, however, by which political parties have substantially affected the functioning of these mixed compensatory systems. Under widespread split-voting, mixed compensatory electoral systems do not lead to quasi-proportional results any more, and instead work similarly to mixed non-compensatory systems. This article explains this strategy of collective split-voting, and shows why possible counter-measures cannot fully prevent the emerging problems.

Mixed compensatory systems are based on a two-part seat allocation process. In the first part, seats are allocated to local candidates in single-seat districts, by plurality or majority rule. The overall number of seats per party is established proportionally, based on their national vote share. In the second part, additional seats for compensatory mandates are allocated to those parties that fall short of the overall number of seats they would need, to win seats in single-seat districts. Usually, voters have two votes, one for a local candidate and one for a compensatory mandate.

This compensation mechanism does not always lead to fully proportional seat allocation, however. Occasionally, a party might win more mandates in single-seat districts than the overall number of mandates it is proportionally entitled to—thereby becoming over-represented, and leading other parties to be under-represented. Such disproportionalities are particularly controversial when produced strategically, as has happened in several elections. In order to maximise their number of seats, rational parties running in mixed compensatory systems may attempt to subvert the proportional compensation mechanism. In several of the instances discussed in this article, split-voting strategies have served this purpose. In such cases, party candidates in the single-seat districts ran under a label that was not on the national party list. As a consequence, the mandates won in the single-seat districts were not subtracted from the number of national proportional mandates, and parties applying this strategy won substantially larger numbers of seats (Elklit, 2008). Similar overrepresentation can be achieved if parties winning single-seat district
seats encourage their voters to cast their party list vote for an allied electoral list (Bochsler, 2010b: 123-129). If a sufficient number of voters adheres to these instructions, it leads to over-representation of the strategically acting parties, and to under-representation of others. To counterbalance such effects, Germany adds extra mandates (Ausgleichsmandate) to the parliament in the case of such seat allocation discrepancies—but this does not resolve the problem, as we will show.

A particularly striking instance of the subversion of PR mechanisms can be seen in the Albanian parliamentary elections of 2005. In these elections, the largest party—the Democratic Party—entered into an informal electoral alliance with a few tiny parties, and instructed its voters to give their proportional votes to these allies. By doing so, the allied parties secured an artificial majority of seats in parliament. In addition to securing a majority of the district mandates for itself, the Democratic Party’s allies won most of the compensation seats. Without such a strategy—i.e., if most PD voters had cast their proportional votes for the PD—the party would not have won any compensation seats, since it was already adequately represented in single-seat districts. As this article argues, Albania represents the first case where such a split-voting strategy was used on a large-scale, in a way that could not be prevented by special provisions in the electoral law, and that resulted in an artificial governing majority.

Collective strategic split-voting brings several new facets to the literature on mixed compensatory electoral systems. It is a strategy with far-reaching consequences for those systems—consequences that have so far been only perfunctorily discussed. Furthermore, these limitations undermine the common view that mixed compensatory electoral systems have PR-like outcomes, or can even be considered to be PR systems with a personal component (Schoen, 1999: 475; Nohlen, 2004; Shugart & Wattenberg, 2001: 584). The split-voting strategy allows political parties and their voters to undermine the proportionality of the seat allocation in their favour.

This article shows how collective split-voting can be used strategically to increase a political party’s representation. Such strategy is particularly beneficial for parties that win many seats in single-seat districts and in mixed electoral systems with a low share of compensation seats. While this strategic effect usually only applies in cases where a considerable number of voters split their votes uniformly, higher numbers of voters splitting their votes can benefit parties even more. We illustrate our theoretically derived models with several simulations, and with a discussion of the growing number of cases of application—particularly the Albanian 2005 parliamentary elections, which play a peculiar role in the discussion of the manipulability of mixed compensatory electoral systems. All analyses rely on aggregated electoral data.

In sum, mixed compensatory systems only result in a proportional seat allocation if voters and parties refrain from specific strategic behaviour. Because of this, it is fair to state that they are PR systems suitable “only for honest men”⁴, or only for voters who abstain from voting strategically.
This article focuses on a case whose findings can be applied to a large number of other countries. After section one explains the collective split-voting strategy, the second section studies the case of the Albanian 2005 elections, and briefly describes other occurrences of the same phenomenon. Section three introduces quantitative formulas that explain the consequences of collective split-voting. Simulation models in section four show that there is no infallible institutional safeguard against split-vote manipulation.

1 Strategic implications of split-voting

While mixed compensatory systems aim at providing a proportional allocation of seats, this ideal cannot always be fulfilled. Occasionally, the number of seat-winning constituency candidates affiliated with a political party is larger than the number of seats the same party would be entitled to according to its list votes. And since the number of single-seat districts cannot be changed ex-post, there is a representational gap between the two tiers. In most cases, the district mandates that a party has won in excess of its overall proportional stake will be cut from the proportionally entitled number of mandates of other parties, leading to disproportionalities in the seat allocation. In such cases, some mixed compensatory systems would increase the number of compensation mandates (cf. Maier, 2007), but this solution often still fails to produce full proportionality, and is not always possible (see section 4). This characteristic of mixed electoral systems offers incentives for voters and parties to engage in strategic, seat-maximising behaviour.

While strategic voting typically aims at preventing votes from being wasted on unsuccessful parties, the vote-splitting strategy discussed in this article is fundamentally different: Mixed compensatory electoral systems provide a loophole that enables voters to make double use of their votes, putting the compensatory mechanism of the system out of order.

Here is how political parties can make strategic use of this loophole: Party A might advise its supporters to vote for its candidates in the single-seat districts, but cast their party list vote for another, say, competitor B. This might de facto be a political ally of party A, although not formally linked. If a large number of voters follow this strategy, party A will win more single-seat district seats than it is entitled to according to its party list vote share—which is substantially reduced because many of its voters have voted for party B in the PR tier. At the same time, party B wins a substantial number of compensation mandates, profiting from the party list votes of voters from party A. Therefore, the number of seats that parties A and B win will be jointly larger than the number of seats that they might have won independently. Party A might also apply the strategy of putting up a list in the PR tier under a different label, but with candidates who belong to party A.

The collective split-voting strategy analysed here differs from common strategic split-voting of individual voters in its motivation, its collective form, and its consequences for seat allocation.
Various factors might motivate voters not to vote for the same political party with both their votes in two-vote systems; some of them are of a strategic nature. One individually motivated split-voting strategy is in line with the classical psychological effect of electoral systems (Duverger 1951): voters who belong to small parties may vote for a candidate of a larger party in the single-seat district election, expecting their favoured party to win PR mandates, even if its candidate is not among the front-runners in their single-seat district (Von Beyme, 2004: 90-91; Cox, 1997). Such strategic behaviour is aimed at giving one’s vote a useful impact on seat allocation. The strategy respects the idea of equal representation, ‘one man – one vote’; if voting sincerely, supporters of small parties would waste their vote on an unviable candidate, and their vote would lose its impact on the electoral result. Coalition-driven split-voters prefer large parties, but cast their party list vote for a small coalition partner, helping it to cross the electoral threshold and ensure the necessary number of seats for the formation of a coalition government (Jesse, 1988; Roberts, 1988; Gschwend, Johnston, & Pattie, 2003; Schoen, 1999; Shikano, Herrmann, & Thurner, 2009). Either individual or small groups of voters may decide to apply such split-voting strategies. Voters do not need to understand the strategic considerations of split-voting to participate in it; they might do so by accident (Schoen, 1999; Karp, Vowles, Banducci, & Donovan, 2002) or by following the strategic recommendation of other, more knowledgeable agents (Bawn, 1999: 502). Typically, political parties have campaigned for split-voting (Jesse, 1988).

By contrast, collective strategic split-voting, as discussed in this article, requires the coordinated action of a large number of the party’s voters, as it only becomes effective if a certain number of voters split their vote uniformly (see section 3). The motivation to do so is not driven by the goal of equal voting power and equal representation, but rather is aimed at leading to over-representation of the parties that encourage their voters to engage in this strategy. Those voters who apply such a vote-splitting strategy increase the relative impact of their vote on the seat allocation in parliament.

Since collective strategic split-voting requires large groups of voters to act uniformly (regardless of whether they are aware of the tactic’s strategic implications: they may simply be following a voting recommendation). Such behaviour can be readily identified among a group of voters, based on an analysis of aggregated electoral results, rather than identifying individual voters. Different forms of strategic split-voting manifest in different voting behaviour and different aggregated electoral results. Under the most conventional split-voting strategies, which are driven by the psychological electoral system effect, the party list vote corresponds to the sincere party preferences. The largest parties might win additional votes in single-seat districts at the expense of smaller parties. In the case of coalition- and threshold-driven split-voting, the district vote expresses the voters’ sincere preference, but small parties might profit from additional votes if they would otherwise risk falling short of the electoral threshold in the PR tier. Collective split-voting can be
identified by a very large difference in the vote between the two tiers, along with an informal agreement between the electoral lists that share their voters. It likely involves an elite-driven campaign for split-voting. If adhered to by the largest part of the electorate, the strategy would mean that an electoral list that appears to be small in the PR tier becomes one of the largest competitors in the single-seat district tier—a pattern that would be inconsistent with the conventional forms of strategic voting (see table 1). If a small group of voters adheres to collective strategic split-voting, it cannot be directly distinguished from other forms of strategic split-voting.

Table 1
Characterisation of conventional and collective split-voting strategies

<table>
<thead>
<tr>
<th>Group of potential split-voters</th>
<th>Conventional strategic split-voting (main form)</th>
<th>Coalition-driven strategic split-voting</th>
<th>Collective strategic split-voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voters with preference for small parties</td>
<td>Voters with preference for large parties (potential future main government parties)</td>
<td>Voters with preference for parties that win some district mandates (typically: large parties)</td>
<td></td>
</tr>
<tr>
<td>Main motivation</td>
<td>Avoiding wasting district vote on small party</td>
<td>Enabling potential small coalition partner to pass electoral threshold</td>
<td>Increasing the degree of representation</td>
</tr>
<tr>
<td>District vote</td>
<td>Strategic vote for one of the front-running parties</td>
<td>Sincere vote for favourite party</td>
<td>Sincere vote for favourite party</td>
</tr>
<tr>
<td>PR vote</td>
<td>Sincere vote for favourite party</td>
<td>Strategic vote for potential coalition ally</td>
<td>Strategic vote for an ally of favourite party</td>
</tr>
<tr>
<td>What happens if vote is not cast strategically</td>
<td>District vote is wasted on hopeless candidate</td>
<td>Small party (possible coalition ally) fails to enter parliament</td>
<td>Voter gets represented as any other voter</td>
</tr>
<tr>
<td>Indicators of the strategy in aggregated voting results</td>
<td>Small parties wins more PR votes than district votes; large party wins slightly fewer PR votes than district votes</td>
<td>Small party, with electoral result around the threshold, wins more PR votes than district votes; large party wins slightly fewer PR votes than district votes</td>
<td>Large party in the district tier wins only a small number of PR votes</td>
</tr>
</tbody>
</table>

2 **Split-voting in Albania, Italy, Lesotho, and Venezuela**

Several cases of collective split-voting demonstrate the relevance of this loophole in mixed compensatory electoral systems. This section presents empirical field evidence, which will subsequently help us show the relevance and practicability of the theoretical models to be elaborated in section three. The Albanian 2005 election stands as the best example of the strategy being applied. The results are readily transferable to other countries.

In Italy, a split-voting strategy was applied in 2001, under the term “liste civette” (literally: owl lists) (Ignazi, 2002; Katz, 2006: 296). Both the centre-left and the centre-right alliance ran part of their constituency candidates in some of the regions on clone lists, under the title “Paese nuovo” (new country) in the case of the centre-left (Ulivo), and under the provocative title “Abolizione scorporo” (abolition of the compensation system) in the case of the centre-right (casa delle libertà). This helped to eliminate part of the compensation effect of the Italian *scorporo* system. The
*scorporo* can be translated as ‘negative vote transfer system,’ and its outcomes are slightly different from other, common mixed compensatory systems (Ferrara & Herron, 2005: 22).

In Venezuela (in 2005) and Lesotho (in 2007), a common form of mixed compensatory system was employed. Major parties ran under two different labels, known in Venezuela under the term 'las Morochas' (the twins). In order to avoid the compensatory mechanism, the ruling party, Movement for the 5th Republic (MVR), was running with its own party list in the PR part of the elections, but its candidates ran under the label Union Electoral Victors (UVE), a formally different twin party in the single-seat districts. UVE candidates won 85.5% of the votes in the single-seat constituencies, but did not contest the PR elections. Therefore, the mandates won in single-seat districts could not be deducted from the MVR’s party tier seats.9

In Lesotho, 2007, the large political parties used the same strategy. Votes and seats in the single-seat districts were essentially divided between the Lesotho Congress for Democracy (LCD) (52.6% of the votes, 62 out of 80 seats), and the All Basotho Convention (28.4% votes, 17 seats). The two parties did not run in the PR tier. Instead, their twin parties, the National Independent Party (NIP) and the Lesotho Workers' Party (LWP), won most of the votes (51.8% and 24.3%) and the seats (21 and 10 out of 40) in the PR tier.10

Both in Lesotho and in Venezuela, the largest parties (those winning most district mandates) split their lists, using one party label for their district candidates and a different label for their PR lists. Therefore, voters did even not have the possibility of voting for the same party in both tiers. Such a strategy can easily be prevented through a minor modification of the electoral rules (see section 3).

By contrast, in the 2005 Albanian elections, the parties ran with lists under the same name in both parts of the electoral system, yet still managed to implement their collective split-voting strategy almost perfectly. Since the Albanian strategy cannot easily be prevented by changes in legislation, and because it occurred under a commonly used mixed compensatory system, it is the most important case in our discussion.

Albania’s post-communist party system has consisted of two main parties: the Socialists (PS) and the Democratic party (PD). Smaller parties include the *de-facto* Greek minority party (the Union for Human Rights PBDNJ) and the Socialist Alliance for Integration (LSI). After the 1997 elections (which functioned under a mixed non-compensatory system) led to huge disproportionalities in favour of the Socialist party, the country switched to a mixed compensatory system in 2001. The new system included 100 single-seat districts, elected by two-round majority vote, with 40 compensation mandates. These were allocated by the largest remainder method with a national 2.5% threshold (4% for coalitions). In 2005, a plurality rule was introduced for the single-seat district elections, and the second election round was abandoned.
Limited to one electoral district, the collective split-voting strategy was first applied in the 2001 parliamentary elections. In the electoral constituency of Dushk, the elections were postponed from 24 June to 8 July for reasons that remain unclear (OSCE 2001). However, the preliminary results of the first round of elections held in all other constituencies on 24 June showed a clear victory for the Socialist Party, with 67 district mandates won, and expectations for acquiring more districts seats in the second round. Since the Socialist Party would only be entitled at most to 67 party list seats, according to the June 24 results, no further party list votes that the Socialists might gain in the Dushk district could produce any compensation mandates. Because of this, the party advised its voters in the Dushk district to cast their party list votes in the by-election for one of three small allies of the party (OSCE 2001). All three Socialist allies had failed to pass the 2.5% national threshold for the allocation of compensation mandates in the regular elections of 24 June. However, thanks to the votes obtained from Socialist voters in the by-election in Dushk, all three parties narrowly passed the legal threshold and won compensation mandates. Split-voting in favour of the small parties followed two strategies: it secured the entry of a coalition partner, and it also worked to garner surplus mandates for the Socialist party—thereby increasing the over-representation of the alliance in parliament.

In the 2005 elections, the Dushk split-voting strategy was employed on a large scale, and its implications went far beyond securing small allies in order to pass a threshold. First, the Socialist party decided to attempt a split-voting campaign across all of Albania. The party allied informally with four small lists – all of them part of the incumbent government coalition – and encouraged its voters to cast their party list vote for one of them (OSCE 2005b: 5-7). The approach worked; there was a wide gap between the Socialists’ percentage of the votes in the single-seat districts (39%) and those cast for the PR lists (9%). Thus, the Socialists won 42 out of 100 district mandates, while their votes would only have been worth some 14 out of the 140 mandates in parliament, if calculated according to PR rules. If a party wins more district mandates than it would have received based on its party list votes, it is allocated surplus mandates – in this case a large number. The four Socialists’ allies, previously all irrelevant players in the Albanian party landscape, won 30% of the party list votes, mainly thanks to voters who voted for the Socialist candidates in the districts. These party list votes entitled them to 16 compensation seats (table 2). Jointly, this secured the Socialists and their allies many more mandates than if Socialist voters had cast their PR vote for the Socialist list.

The Socialists were not the only player to adopt such a split-voting strategy. Realising that the split-voting strategy might earn the Socialists a majority of the seats in parliament—even with only a minority of votes—the Democratic Party (PD) decided to use the same strategy. For its part, the Democratic Party cooperated with an alliance of seven minor parties and encouraged its supporters to cast their party list votes for one of them (OSCE 2005a). All
seven allies had previously been negligible (some ran on the PD list in 2001, and the others won jointly less than 2% of the national PR vote). The candidates of the PD won 44% of the votes in the single-seat districts and 56 district mandates, but the party secured only 8% of the PR votes, while its tiny allies won 33% of the PR votes. This split-voting strategy secured the PD’s allies 18 compensation seats, which, jointly with the PD’s district mandates, resulted in a 74-seat majority in the 140-seat parliament. This would not have occurred without the strategy just described. One of the PD allies, the previously marginal Republican Party, earned 20% of the party list votes and nominally became the largest party in Albania by relying on the strategic votes.

While the two large parties and their allies were able to increase their parliamentary representation, two non-allied small parties, who had 12.5% of the votes, paid the price of the split-voting strategy: the Socialist Alliance for Integration (LSI) and the de-facto Greek minority party (PBDNJ). In the absence of split-voting, they would have won 18 compensation seats (plus a district mandate), and become the pivotal players in parliament, since neither of the two large parties could have won an absolute majority of the seats. Given the wide-scale split-voting strategy, however, massive surplus mandates were created for the large parties, thanks to the compensation seats given to the large parties’ allies; the share of the compensation seats of the non-allied small parties shrank from 18 to 7.

Table 2
Electoral results in Albania 2005; single-member districts (left) and proportional results (right); parties are assembled by (informal) party blocks. Source: Central Electoral commission, own calculations.

<table>
<thead>
<tr>
<th>Party Block &quot;Partia Demokratike&quot;</th>
<th>District votes</th>
<th>District seats</th>
<th>PR vote</th>
<th>PR seats</th>
<th>Total seats</th>
<th>Seat share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>44.0%</td>
<td>56</td>
<td>41.1%</td>
<td>18</td>
<td>74</td>
<td>52.9%</td>
</tr>
<tr>
<td>PD</td>
<td>44.0%</td>
<td>56</td>
<td>7.7%</td>
<td>0</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>ALDM - BDSH</td>
<td>0.0%</td>
<td>0</td>
<td>0.5%</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>ALDM - BLD</td>
<td>0.0%</td>
<td>0</td>
<td>1.1%</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>ALDM - LDLNJ</td>
<td>0.0%</td>
<td>0</td>
<td>0.7%</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>ALDM - PBKD</td>
<td>0.0%</td>
<td>0</td>
<td>0.6%</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>ALDM - PDK</td>
<td>0.0%</td>
<td>0</td>
<td>3.3%</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>ALDM - PDR</td>
<td>0.0%</td>
<td>0</td>
<td>7.4%</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>ALDM - PR</td>
<td>0.0%</td>
<td>0</td>
<td>20.0%</td>
<td>11</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Party Block &quot;Partia Socialiste&quot;</th>
<th>District votes</th>
<th>District seats</th>
<th>PR vote</th>
<th>PR seats</th>
<th>Total seats</th>
<th>Seat share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>42.2%</td>
<td>42</td>
<td>37.2%</td>
<td>16</td>
<td>58</td>
<td>41.4%</td>
</tr>
<tr>
<td>PSSH</td>
<td>39.4%</td>
<td>42</td>
<td>8.9%</td>
<td>0</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>0.8%</td>
<td>0</td>
<td>4.8%</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>PDSSSH</td>
<td>0.6%</td>
<td>0</td>
<td>4.2%</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>PAA</td>
<td>0.7%</td>
<td>0</td>
<td>6.6%</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>PSD</td>
<td>1.3%</td>
<td>0</td>
<td>12.7%</td>
<td>7</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Small parties over the 2.5%/4% threshold (4% for coalitions)</th>
<th>District votes</th>
<th>PR vote</th>
<th>PR seats</th>
<th>Total seats</th>
<th>Seat share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>9.2%</td>
<td>12.5%</td>
<td>6</td>
<td>7</td>
<td>5.0%</td>
</tr>
<tr>
<td>LSI</td>
<td>8.3%</td>
<td>8.4%</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>PBDNJ</td>
<td>0.9%</td>
<td>4.1%</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Small parties below the 2.5%/4% threshold</th>
<th>District votes</th>
<th>PR vote</th>
<th>PR seats</th>
<th>Total seats</th>
<th>Seat share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.9%</td>
<td>9.1%</td>
<td>0</td>
<td>1</td>
<td>0.7%</td>
</tr>
</tbody>
</table>
Such disciplined behaviour on the part of large groups of voters – 79% of the large party voters split their vote according to the collective strategy – might be astonishing, especially in a young democracy, given the complex voting strategy. Detailed information on the electoral campaign remains largely inaccessible, and is sometimes contradictory. Reports and expert interviews reveal, however, that the pre-election agreements were implemented by different means, and the voting results reveal that the strategy worked almost perfectly (OSCE ODIHR, 2005a, 2005b). The major parties asked their supporters to vote for the minor allied parties in the party list vote. This message was spread through campaign material, media, and direct contacts with voters. This strategy was crucial in achieving such widespread and systematic vote splitting.

By implementing alliances with minor parties, the major parties ultimately secured themselves the intended gains of their electoral strategy. Thirty of the Republican Party PR electoral list candidates were members of the Democratic Party; some of them were even members of the incumbent parliament (OSCE ODIHR, 2005a: 10).

3 A generalised model of split-voting and number of compensation seats

All of this suggests the question: To what extent might legal provisions prevent parties from applying such collective split-voting strategies, or prevent the disproportional consequences of collective split-voting? Minor amendments to the electoral legislation can force parties to present their own lists in the PR tier and can hinder technically independent candidates from running in single-seat districts. Under German electoral law, voters cast both their votes on the same ballot paper. A voter’s party list vote is not counted the voter gives her constituency vote to an independent district candidate or to a candidate of a party that has no party list in the PR tier. The Albanian strategy appears more difficult to prevent, however. The German Constitutional Court has ruled that surplus mandates are legal (on the grounds that they are so rare) it has not proposed a solution to prevent the consequences of collective split-voting.22

To avoid the disproportionalities resulting from collective split-voting, scholars, experts and the German Court have discussed the possibility of varying the number of mandates in parliament (OSCE ODIHR, 2005a: 5). Albania established such a provision in its 1992 electoral law (Commission on Security and Cooperation in Europe, 1992: 6), and several German Bundesländer have applied similar ones (Maier, 2007). This section develops a general model to calculate the size of parliament needed to compensate for the effects of split-voting. Results suggest that varying the size of parliament is not effective in cases where a large proportion of voters split their vote, since this would require an inordinate number of additional seats.
Surplus mandates are achieved when a party wins more district mandates than it would be entitled to according to its party list vote share, and the result is its over-representation. This allows us to predict the consequences of uniform split-voting on seat allocation. Surplus mandates are produced when the party list vote share of a party is low enough that the party wins more district mandates than the number of mandates it is entitled to proportionally. Naturally, a fully proportional seat allocation is almost never achieved, since some rounding is required in every PR electoral system. For this study, ‘disproportional’ seat allocations are defined as the distributions that deviate from the allocation if all parliamentary seats were elected according to the PR rules that apply for the allocation of compensatory mandates, considering all legal thresholds and electoral districts.

Surplus mandates for party $i$ appear (in larger numbers) if party $i$ wins many seats in single-seat districts $s_{i,SSD}$, especially in fragmented party systems where only a low vote share $v_{i,SSD}$ is needed to win district mandates.\textsuperscript{23} In addition, surplus mandates occur if voters of party $i$ cast many of their party list votes for other parties (i.e., if the split-voting quota $d_{i,SSD}$ is negative), if there are few wasted votes $w_{PR}$, and if the electoral system has a low ratio of compensatory seats to single-seat districts ($S_{comp}/S_{SSD}$). Since the ratio of single-seat district seats won to overall district votes is typically only high for the largest political parties in a party system, these parties are most likely to benefit from surplus mandates and to enjoy the highest payouts from collective split-voting strategies. For formula derivations, please see the appendix.

\begin{equation}
S_{i,SSD} > \frac{v_{i,SSD} \cdot (1 + d_{i,SSD})}{1 - w_{PR}} \cdot \left( \frac{S_{comp}}{S_{SSD}} + 1 \right)
\end{equation} 

Surplus mandates lead to the overrepresentation of party $i$, and a diminished number of seats for other parties (see appendix). Two of the variables in formula 2 are directly affected by strategic behaviour and by institutional design. Parties can campaign for their voters to increase split-voting $d_{i,SSD}$ to the extent that surplus mandates are created, and lawmakers can increase the size of parliament so that the ratio of compensation mandates to single-seat districts becomes large enough to prevent the creation of surplus mandates.

Problems evolving from split-voting might sometimes be resolved by having parliaments with flexible numbers of mandates. Formula 3 establishes the number of compensation mandates needed to prevent the creation of surplus mandates and, hence, to avoid the consequences of split-voting.

\begin{equation}
S_{comp} \geq S_{SSD} \cdot \left( \frac{s_{i,SSD} \cdot (1 - w_{PR})}{v_{i,SSD} \cdot (1 + d_{i,SSD})} - 1 \right)
\end{equation}

For high incidences of split-voting, the number of compensatory seats would rise to very high numbers, and thus be politically infeasible. If 80% of the voters of party $i$ split their votes
\( (d_{i, SSD} = -0.8), \) and \( s_{i, SSD} \approx v_{i, SSD} \), the parliament would need to count five times more seats than single-seat districts. Parliaments elected by mixed compensatory systems often consist of at least 50% district mandates – meaning that, in such a case, the parliamentary seat number would need to increase by 150%.

The required number of seats might even outnumber the number of eligible voters once a party (even a tiny one) wins at least one district seat, but gains only a few PR votes. Roughly (disregarding wasted votes and rounding effects), this is the case if any party wins more district seats than it gets votes in the PR tier (in absolute numbers). The PR votes are the baseline for the overall proportional seat allocation, and therefore, this party would hold more seats in the parliament than it obtained votes. Given that the representation ratio (seats per votes) should be the same for all parties, the parliament would count more members than there are eligible voters.

In the special case where party \( i \) wins single-seat districts, but does not win a single PR vote \( (d_{i, SSD} = -1) \) – the cases of Lesotho and Venezuela – there is no solution to the equation. Thus, if most of the voters of any party split their vote, varying the number of seats in parliament would not appear to be a solution that allows for proportional results.

The only viable solution to assure a quasi-proportional seat allocation (preventing the potential of split-voting strategies, even if they are applied at a highly consequential level) might be to move to a one-vote mixed electoral system, where the district vote is always linked to a political party in the PR tier. This, however, eliminates many of the original advantages of mixed compensatory systems.24

4 Simulating the consequences of collective split-voting

To illustrate the consequences of different degrees of split-voting, I have run two simulations based on the results of the Albanian 2005 elections and three elections in Lesotho and New Zealand.25. First, I have simulated the seat allocation for parties for different sizes of parliament, with roughly 120 to 200 seats (in each case including the original size of parliament), varying the degree of split-voting for the two large parties from -1 to 0.26 At 0, all parties win the same number of party list votes as single-seat district votes, and at -1, all large party voters cast their party list vote for an ally of the large party. The fictitious seat allocation was calculated using the Hare/Niemayer (largest-reminder) formula, which applies in Albania. The simulation is based on the assumption that single-seat district mandates are won by a large party, which is largely true in the Albanian case. Single-seat district mandates are safe, and hence not affected by other parties' split-voting strategies. Therefore, if this assumption does not hold, the simulations slightly over-estimate the effect of split-voting.
Figure 1 shows how systematic split-voting by the large political parties will affect the electoral results of the small parties. The vertical axis displays the representation ratio, or the rate of seat shares per vote shares, for single parties, which is proportional to their number of seats. Here, we took the average rate for all small parties. Values above 1 are over-proportional; below 1, the parties are under-represented. At low degrees of split-voting, small parties win a slightly higher overall seat share than their vote share amounts to. In Albania, the ratio of the vote share to seat share (representation ratio, or A-ratio) of the small parties (LSI and PBDNJ) amounts to approximately 1.09, hence they are slightly over-represented, as are all the other parties that pass the 2.5% electoral threshold. An increased number of voters giving their party list vote to an ally of the large parties (negative quota of split-voting) impacts the number of seats allocated to small parties, so that their representation ratio falls. In parliaments with fewer compensation seats (here, 120 seats, denoted with + symbol), the representation ratio falls even more, and at lower levels of split-voting than in larger parliaments.

If all large party voters split their vote strategically, small parties would lose most of their seats; instead of a representation ratio of 1.09 (with no split-voting), there would be a representation ratio of either 0.2 (120-seat-parliament), 0.34 (140-seat-parliament) or of 0.56 (200-seat-parliament). Thus, such a split-voting strategy might cost small parties a very substantial portion of their seats (depending on the size of parliament)— or, as suggested in the previous section, these small parties’ seat share would be lowered by the number of surplus mandates $s_X$, according to the size of parliament $S_T$. In the case of substantial split-voting, the resulting seat allocation proves equal to the one that results in mixed non-compensatory (parallel) systems; hence, large parties completely disable the compensation mechanism of the mixed compensatory systems. The cases of the 2005 elections in New Zealand and the 2007 elections in Lesotho show the two extremes: in one case, there was a very low degree of split-voting, and the two largest parties won almost the same amount of votes for their PR lists and their district candidates. In the other case, the two largest parties did not have any PR list at all, hence split-voting reached its maximum. Simulation results are not that close to reality in the case of the New Zealand 2008 elections, since several further aspects produced slight advantages of small parties over larger parties. One involved positive discrimination measures in favour of the Maori minority, so that the effect would not play for this party. Two other small parties (ACT New Zealand and the Green party) profited from getting 24% and 22% more votes on the PR lists than in single-seat districts. This blurred their representation ratio (which refers to their district votes) in the actual situation, but would make them even more sensitive to split-voting by other parties.
Figure 1a-1d. Collective split-voting (X-axis), on the electoral success of small parties (representation ratio, Y-axis). Simulation for three sizes of parliament, and for real values for four national elections (Albania, 2005; Lesotho, 2007; New Zealand, 2005, 2008).

The second simulation illustrates the degree to which the number of mandates must be increased in a hypothetical, flexibilised parliament, in order to provide for proportional outcomes if large parties apply a wide-scale split-voting strategy. A flexible parliament size (i.e., one that provides for a flexible number of compensation mandates), would eliminate the manipulative potential for split-voting strategies, since the overall result would always be fully proportional. Our simulation
estimates how many parliament seats would be needed to avoid overrepresentation of large parties for different degrees of split-voting. Figure 2 shows the simulation based on the Albanian 2005 election results for single-seat districts, and uses split-voting quotas ranging from –1 (all party list votes of large parties go to allied parties) to 0 (no split-voting). For other countries, the same pattern results.

![Graph showing split-voting and required number of seats in parliament](image)

**Figure 2:** Split-voting and required number of seats in parliament, if the number of compensation seats is proportional.

*Simulation, on the basis of the vote and seat distribution in the district tier in the Albanian elections 2005. The split-voting quota shows the relative difference of votes cast for large party in the single-seat districts (basis) and in the PR tier.*

At low levels of split-voting, the increase of mandates needed for full compensation proves moderate. But at high levels, the required number of seats grows very large. In the case of the 2005 Albanian elections (which had a 79% incidence of vote splitting), a parliament of almost 600 seats would have been necessary for a proportional allocation of seats, compared to parliament’s actual size of 140. Thus, this reform would not be politically feasible. Compromises between the ideal of proportionality and the necessity of a limited size parliament are conceivable. However, any compromise would still reward parties that win surplus mandates with over-representation, while other parties would be net losers.
5 Conclusions

This article highlights the mixed compensatory electoral system’s potential limitations for providing proportional seat allocation. Often, mixed compensatory systems are thought of as equivalent to PR—though with a stronger personalised element—and are thought to lead to proportional results (Nohlen, 2004: 190; Schoen, 1999: 475; Shugart & Wattenberg, 2001: 584).28 I show that the proportional character of electoral outcomes can be circumvented by collective strategic vote splitting.

If a substantial part of the electorate of a large party splits its votes uniformly—giving the district vote to one party and the party list vote to a different, allied party—then both allies can win substantially more seats than if there were no such split-voting in place. Hence, large parties that win seats in both tiers can profit, winning additional seats at the expense of small parties, which are deprived of a comparable compensation mechanism. The disproportionalities that result from this tactic are particularly controversial when they are strategically produced.

The possibility of a wide-scale application of such strategy might appear unrealistic, but a growing number of cases show that it can, indeed occur—not just in Albania, but also in Italy, Venezuela, and Lesotho. In the 2005 Albanian parliamentary elections, the major political parties encouraged supporters to cast their district ballots for their candidates and their PR ballots for a small allied party. The highly effective implementation of this strategy—which 79% of the large parties’ voters following it—substantially affected seat allocation in the Albanian elections, and enabled a Democratic Party-based coalition to win an artificial majority of seats in parliament. This deprived the small parties of 60% of their seats, and of their role as the pivotal voter in parliament. If all major parties’ voters split their vote in a similarly effective manner, then the outcome of the electoral system would be basically indistinguishable from a mixed non-compensatory system.

Certainly, mixed non-compensatory systems represent a legitimate institution for electing parliaments. It is not problematic per se, if a mixed non-compensatory system is applied for elections. Likewise, split-voting is a possibility inherent in all two-vote mixed electoral systems, and its intended advantages can only materialise if voters are free to cast different votes in each tier of the system. However, individually motivated split-voting—whether intended to give support to a strong personality in a single-seat tier, to guarantee forceful representation of a local constituency, or to avoid wasting a vote on a chanceless candidate—is not the same as the collective split-voting organised by political parties. The latter differs in its political aim, which is giving large political parties an over-representational advantage. Our study has shown that the consequences of split-voting are very different when carried out by large groups, on a large scale.
Mixed compensatory systems were introduced with the intention of attaining quasi-proportional seat allocation. But if large parties adopt collective split-voting strategies, they can achieve substantial over-representation, meaning that seat allocation does not correspond with lawmakers’ intentions for how the system should work. Such strategies are perfectly legal.

Only a few parties – those that win in a substantial number of single-seat districts – can profit from such a collective split-voting strategy. The over-representation that they achieve may lack good faith and appears illegitimate, since their strategy aims at hindering proportional seat allocation. Collective split-voting implies that other parties that do not (or cannot) apply such a strategy are deprived of some seats in parliament (and possibly their role in the governing coalition). This representational imbalance may even approach the point where the parliament is indistinguishable from one elected by a mixed non-compensatory system.

This raises the question of the legitimacy of a governing majority won under a “Dushk” voting strategy. There would be an important difference between identical outcomes produced by a mixed non-compensatory system and by mixed compensatory electoral rules with strategic split-voting. Parliamentary majorities that rely on a minority of votes, which are produced by a mixed non-compensatory electoral system, are legitimate, because they have been won according to a legitimate electoral system. By contrast, parties that win seats by applying a collective split-voting strategy involves strategic use of electoral institutions that distorts the proper functioning of the electoral system. In contrast to common forms of strategic voting, collective strategic voting is not aimed at saving a vote from being wasted, but instead helps to increase the voting power of some parties’ voters, to enable them to have a stronger impact on the electoral outcome than other voters. If such form of strategic voting, which as others is maximising the political impact of large parties, is used, the move to different electoral systems, where such strategic behaviour becomes obsolete might be advisable. It is thus no wonder that Albania switched to a simple PR system for the 2009 elections. For countries that still use mixed compensatory systems, or intend to introduce them, the question of how to deal with the potential for manipulation via coordinated split-voting remains a challenging one.

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Short bio
Daniel Bochsler is Assistant Professor of Democratisation at NCCR Democracy at the University of Zurich. He received his PhD in Political Science from the University of Geneva, and has been for research stays at the Universities of Tartu, Belgrade, at the University of California at Irvine and at the Central European University in Budapest. His main research interests include political representation, with a special focus on post-communist democracies. His monograph “Territory and Electoral Rules in Post-Communist Democracies” has been published in Palgrave, and his articles have been published, among others, in Democratization, Electoral Studies, and Public Choice.

References


1 The Hungarian system is slightly different from typical mixed compensatory systems, using a positive vote transfer system. Italy dropped a similar negative vote transfer system in 2006. In Scotland and Wales, the regional assemblies are elected by a mixed compensatory system.

2 The idea of the proportional representation through mixed compensatory systems by default assumes that the PR vote is the most important vote that expresses their main preferences.

3 The possibility of such practice has been mentioned earlier by Klingemann and Wessels’ (2001: 286), while Behnke (2003: 56) finds that vote-splitting is one of the driving force of surplus mandates. This has also been pointed on by the Constitutional Court of Germany in its sentence of 10 April 1997, cf. Bundesverfassungsgericht 95, 335, Überhangsmandate.

4 In this, they are quite similarly to the Borda Count, which has become famous as an electoral system “intended only for honest men”. This is how its own father, Jean-Charles de Borda, called it (Black, 1958: 182).

5 Typically, voters might cast their PR vote for their favourite party, but vote for a candidate of a different party in their local district, because of their preferences for a certain candidate (Moser & Scheiner, 2005; Burden, 2009). Brunell and Grofman (2009) introduce the notion of sincere ticket splitting.

6 Other voters might split their vote between two parties that they would like to see to govern in a coalition (Austen-Smith & Banks, 1988; Blais, Aldrich, Indridason, & Levine, 2006; Bawn, 1999: 501-502), or because they want to balance their vote between different competitors (Alesina & Rosenthal, 1995; Fiorina, 1996). Strategic voters may consider the candidate of their favourite party to have low chances of getting elected, and instead give their district vote to one of the major competitors (Fisher, 1973; Cox, 1997; Bawn, 1999; Reed, 1999; Moser & Scheiner, 2005; Herrmann & Pappi, 2008).

7 There are usually no questions included in common surveys that allow the distinction of a collective vote-splitting strategies from other forms of strategic voting. The survey that was conducted for the Albanian 2005 elections did not contain any questions about strategic voting. Also, there were large discrepancies between the voting results and the voting intention expressed in the survey, as apparently, voters rather indicated their sincere preferences rather than their strategic vote (Ilirjani, 2005).
The strategic effect of split-voting occurs only if a large number of voters employ the same strategy uniformly. Hence, the strategy can be best traced if focusing on the parties’ nomination strategies, electoral campaigns, and aggregated voting results.


11 Calculation of the seat allocation based on the results of the PR votes cast in all districts except for Dushk; details available from the author.

12 To be precise, it was not just Socialist voters who helped them in this matter; some 1000 to 1500 out of 3900 voters for the Democratic party candidate in the majority race in Dushk must have voted for the small Socialist allies, as suggested by aggregated vote results.

13 Aleanca Demokratike (AD), Partia Demokracia Sociale e Shqipërisë (PDSSH), Partia Agrare Ambientaliste (PAA) and the Partia Socialdemokrate e Shqipërisë (PSD). The parties were particularly close to the Socialists; among others, they had formed a multi-partisan parliamentary group in the previous legislature, and had competed in 2003 jointly in local elections.

14 For the Socialist-led coalition, the PR votes and the district votes correlate closely across the 100 electoral districts. The same is true for the coalition around PD.

15 Partia Bashkimi Demokrat Shqiptar (PBDSh), Bashkimi Liberal Demokrat (BLD), Lëvizja për të Drejftat dhe Liriti e Njeriut (LDLNJ), Partia Balli Kombëtar Demokrat (PBKD). Partia Demokristiane e Shqipërisë Aleanca për Liri (PDK), Partia Demokrate e Re (PDR), Partia Republikane (PR).

16 While some of the small parties have older roots, and existed since the start of the democratisation process in 1992/1991, they remained irrelevant in the Albanian party system, without significant support and were not substantially institutionalised (Biberaj, 1998: 70, 235).

In the 1992 elections, where the Democratic party, the Republican Party, and the Social Democratic Party were locked in opposition against the Socialists, the Republican Party competed with a program close to the Democratic Party’s, but some Republicans still saw the Democrats as their main rivals. In districts where the Socialists did not have real chances to get elected, the Republicans campaigned against the Democrats. The Social Democratic Party was close to the Socialist Party (Biberaj, 1998: 133).

17 Certain parties failed to pass the 2.5% threshold for political parties, but jointly they passed the 4% threshold for coalitions, so that all 33% of the votes were considered in the allocation of compensation seats.

18 Even if the Republican Party’s history goes back to the democratisation in 1990 (Biberaj, 1998: 70,133), it was previously a marginal party and an ally of the Democratic Party.

19 My calculation is based on the votes from the PR tier, but counting the allied party blocs (as listed in table 2) as a whole: PD+allies 41.1% votes – 63 seats. PSSH+allies 37.2% votes – 57 seats. LSI 8.4% votes – 13 seats. PBDSh 4.1% votes – 6 seats. In the previous elections, 2001, indeed the PD engaged in such a party alliance in the PR tier with some of the parties that in 2005 were included into the PD bloc.

20 As shown below, a more perfect vote splitting would have even slightly increased the impact.

21 Albanian parties tried in 2001 to register 112 candidates as "independents" in order to avoid the compensation mechanism. This attempt was not successful, as the Electoral Commission ruled it out (OSCE ODIHR, 2001: 8-9).

22 Bundesverfassungsgericht 95, 335, Überhangsmandate, sentence of 10 April 1997, C IV

23 This can result from a highly fractionalised party system, from malapportionment (a party that is particularly strong in small constituencies), differences in turnout between constituencies (in the districts held by one party, turnout is lower), and from a specific distribution of votes across constituencies (i.e., if a party wins narrowly in some constituencies, but collects only very low vote shares in others).

24 Still, one-vote-systems allow similar strategies, even if to a lower degree. If parties anticipate that a candidate is likely to be elected with a low vote share in her single-seat districts, they might nominate her as an independent candidate, and increase the number of seats won to a disproportional high seat share. This tactic can be hindered by positive vote transfer systems (as are currently in use in Hungary), with a single round of district elections (Bochsler, 2010a). In the positive vote transfer system, votes for non-elected constituency candidates are transferred to the compensation tier, and based on these unused votes, compensation mandates are allocated proportionally. One-vote and positive vote transfer systems abolish other features that are considered to be positive in two-vote-systems, such as the ability to choose personality and party with two distinct votes. One-vote mixed compensatory systems are applied in Romania, in South Korea, and for the regional elections in two German Bundesländer (Nordrhein-Westfalen and Baden-Württemberg, cf. Müller (2004)).

25 Only cases with a mixed compensatory system with no special features, such as regional districts in the PR tier, and where detailed results (including seat and vote allocation in each of both tiers) are easily available. Data: national election commissions.

26 The simulation relies on the results of the 2005 elections for the single-seat districts. In this simulation, the split-voting quota is equal for both large parties.
The parliament is large enough if each party, based on its PR vote share, is entitled to at least the number of mandates as its number of single-seat districts. Hence, the simulation takes the number of single-seat districts as a given, and calculates the number of parliamentary mandates needed. Under the Sainte-Laguë PR formula, we can easily calculate the size that a parliament needs to be, to allocate a required number of mandates to a party with a given vote share. Other parties are assumed to be above the 2.5% threshold in the PR tier. For each party \( j \) with a vote share \( v_j \), one can calculate how many mandates \( S_j \) the party would win under the PR rule, after establishing the number parliamentary seats \( S \). As a rule of the thumb, results with a decimal remainder below 0.5 are rounded down, and those with a decimal remainder above 0.5 are rounded up.

\[
S_j = S \cdot v_j \pm 0.5
\]

This can be transformed, in order to estimate how many seats in parliament \( S \) are needed in order to guarantee that party \( j \) wins at least the number of seats \( s_j \) that it wins in the single-seat districts. As \( S \) needs to be an integer, the result of the formula needs to be rounded up to the next larger integer.

\[
S \geq \frac{(S_j - 0.5)}{v_j \cdot (1 - d)}
\]

The seat allocation under the largest remainder formula applied in Albania differs only marginally, and in rare cases, from the Sainte-Laguë formula, so the differences are negligible.

At least if there is a sufficiently large number of compensatory mandates (Moser & Scheiner, 2004: 580; Cox & Schoppe, 2002: 1029; Behnke, 2003; Bochsler, 2007).
Daniel Bochsler

A quasi-proportional electoral system "only for honest men"? The hidden potential for manipulating mixed compensatory electoral systems – Appendix

Appendix: Theoretical derivation of the formulas

The collective split-voting strategy implies that a party wins district mandates, but only few PR votes, which may produce surplus mandates. These are strategically produced, if parties call for their voters to split their votes, with the intention to reduce their PR vote share. Surplus mandates and disproportional outcomes result if the PR vote share of a party is low enough, so that it wins more district mandates than the number of mandates it is entitled to proportionally.

Earlier studies have established empirical estimates of the factors affecting the creation of surplus mandates for the specific German case (Behnke, 2003, Behnke et al., 2003). My model focuses particularly on the impact of split-voting, and is based on a logical model, which provides estimates on solely theoretical grounds and is tested through simulations in section 4.¹

I estimate the absolute number of seats that party \( i \) is entitled (\( s_{i,PR} \)) to, according to its PR vote share, proportionally to these votes (\( v_{i,PR} \)),² and is affected further by the overall PR vote share cast for parties below legal thresholds of representation (\( w_{PR} \)). Out of \( S_T \) seats in parliament, party \( i \) is entitled to a seat share of \( s_{i,PR} = \frac{v_{i,PR}}{1 - w_{PR}} \) seats. In the single-seat district tier, the same party \( i \) wins a seat share of \( s_{i,SSD} \) seats, out of overall \( S_{SSD} \) seats. The seat share in the single-seat districts relies on the distribution of votes of all parties across districts. Further, we consider that the total number of seats in parliament \( S_T \) is the sum of the number of single-seat district mandates \( S_{SSD} \) and the number of compensation mandates \( S_{comp} \). Surplus created are created if party \( i \) wins more single-seat district mandates than it would be entitled proportionally, or if the following condition applies:

\[
S_{i,SSD} > \frac{v_{i,PR}}{1 - w_{PR}} \left( \frac{S_{comp}}{S_{SSD}} + 1 \right)
\]

(1)

The number of surplus mandates \( s_X \) is defined for each party as the number of single-seat district seats minus the number of proportionally entitled seats (if it results to be positive), or added up over

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¹ Occasionally, especially if there are few compensation mandates, surplus mandates might even be created without any vote-splitting. This was the case for instance in German Bundesländer that employ a one-vote system for the regional elections. In such cases, vote-splitting would further increase the number of surplus mandates.

² This estimation is simplified, as it neglects rounding differences emanating from the employed PR formula and from electoral districts that might apply in the PR tier.
all parties as $s_X = \sum_{i=1}^{n} \min \left( 0, s_{i,SSD} \cdot S_{SSD} - \frac{v_{i,PR}}{1 - w_{PR}} \cdot S_T \right)$. If a party winning single-seat district seats decreases its vote share $v_{i,PR}$ in the PR part, the right-hand side of formula 1 decreases, and below a certain level, the party wins surplus mandates. This is the rational behind the split-voting strategy described for the Albanian case. If surplus mandates are created, and not compensated for through an increased size of parliament, this means that a lower number of mandates is available for distribution on the parties not winning surplus mandates. Hence, the parties get underrepresented to the degree that the overall number of available seats is diminished through the surplus mandates: if $S_{i,PR} = \frac{v_{i,PR}}{1 - w_{PR}} \cdot S_T$ would be the proportional number of mandates, after deduction of surplus mandates, it results at $S_{i,PR} = \frac{v_{i,PR}}{1 - w_{PR}} \cdot (S_T - S_X)$, so that their seat share gets lowered by $S_X/S_T$.

A simple transformation of the formula further shows how many voters need to split their vote so that surplus mandates are created. The split-voting quota $d_{i,SSD}$ compares party $i$’s share of PR votes to its share of the district votes $d_{i,SSD} = \frac{v_{i,PR}}{v_{i,SSD}} - 1$. It is 0, if the party’s vote share is equal in both tiers (which – for sure – does not mean that there were no voters who cast their vote for different parties, but that they cancelled each other out). Negative values indicate the percentage rate that a party obtains fewer PR votes than district votes, while positive values indicate the rate that a party wins more PR votes than district votes. Hence, we can express the PR votes in single-seat district votes and the degree of split-voting.

$$s_{i,SSD} > \frac{v_{i,SSD} \cdot (1 + d_{i,SSD})}{1 - w_{PR}} \cdot \left( \frac{S_{comp}}{S_{SSD}} + 1 \right)$$ (2)

Resolving this formula to $d_{i,SSD}$, we see that each party that wins single-seat district mandates can benefit from surplus mandates, if its voters split their votes systematically enough (if split-voting quota becomes sufficiently negative).

$$d_{i,SSD} < \frac{s_{i,SSD} \cdot S_{SSD} \cdot (1 - w_{PR})}{S_T \cdot v_{i,SSD}} - 1$$

Given a certain number of split-votes, we can derive the number of mandates necessary to allow for full compensation.
$$S_{comp} \geq S_{SSD} \cdot \left( \frac{s_{i,SSD} \cdot (1 - w_{PR})}{v_{i,SSD} \cdot (1 + d_{i,SSD})} - 1 \right)$$ (3)