Which mixed-member proportional electoral formula fits you best?
Assessing the proportionality principle of positive vote transfer systems

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Mixed-member proportional systems (MMP) are a family of electoral systems which combine district-based elections with a proportional seat allocation. Positive vote transfer systems belong to this family. This paper explains why they might be better than their siblings, and examines under which conditions full proportionality is reached. A formal model shows that full proportionality depends on the number of compensation mandates available, and on the degree of coordination among voters and parties. The model is applied to six elections in Hungary.

Keywords: Electoral systems; mixed-member proportional; positive vote transfer; proportionality; Hungary.

1 Introduction

There is fascination both among political practitioners and among academics for electoral systems which mix elements of direct, personality-centred elections with a proportional seat allocation in parliament (Shugart and Wattenberg 2001a: 582-3, Shugart 2013). Most of the attention is devoted to mixed-member proportional electoral systems (MMP)\(^1\), while their close relatives, positive vote transfer systems – practiced in Italy and Hungary\(^–\), are treated, at best, as a curiosity in the electoral system family.

The differences between the MMP and the positive vote transfer system have rarely been addressed in the comparative literature. Two of its features make the positive vote transfer formula a compelling alternative to commonly used MMP rules: it is not only simple and more intuitive than common MMP rules, but it is also immune to strategic manipulation, which can occur under MMP rules. However, the key feature of MMP systems – their compensation mechanism, which allows for a seat allocation in proportion to the vote shares of parties – has not been scrutinised for positive vote transfer systems. This paper develops a model which explains in which situations positive vote transfer formulas will lead to a proportional seat allocation, and tests it empirically, relying on the results of six elections in Hungary.

Positive vote transfer formulas provide for a seat allocation in two tiers, with one part of the seats being allocated in electoral districts by plurality or majority rule. A second tier provides for proportional compensatory mandates. The Italo-Hungarian version of the MMP formula is certainly easier to grasp than that of common MMP types. Every vote counts once: either it helps a candidate to become
elected in his or her electoral district, or, if candidates fail to become elected, their votes are transferred to their political parties, and the party lists are allocated seats on a proportional basis, based on these transferred votes. Certainly, when Italy reformed the electoral system of its Senate in 2005 (D’Alimonte 2005), and Hungary and became the only case of application, the reputation of the positive vote transfer system suffered. In the Hungarian case, the positive vote transfer mechanism is only one element of the electoral system, embedded in a complex conglomerate of aspects of different electoral rules (see section 5 of this paper), which make it one of the most complex systems worldwide. When referring to the positive vote transfer system, we discuss only one aspect – the proportional compensation aspect of the Hungarian electoral rules.

In contrast to MMP systems, positive vote transfer formulas are less prone to voting strategies. MMP systems might encourage political parties to adopt complex strategies of split-voting, which will increase their representation in parliament. This has lead to representational paradoxes (see below), and it has undermined the legitimacy of MMP systems in a number of countries including Albania, Lesotho, Romania, and Venezuela (Bochsler 2012). Hungary is one of the few cases where a young democracy has used MMP rules (or a sub-type thereof) over a longer time period, and no such strategic games have emerged. Under the positive vote transfer sub-type, such strategies are not fruitful.

Ruling out party strategies which can undermine the proportional character of the system, the positive vote transfer system might thus be considered the only truly proportional type of MMP system. This paper concludes that their capacity for delivering a proportional seat allocation depends not only on the number of compensation seats available, but also on the political behaviour of voters and parties. On the one hand, the number of compensation seats needed to provide for a proportional seat allocation varies widely, and depends essentially on the vote shares cast for the largest parties in the single-seat districts. On the other hand, the very same electoral rule can in the situation of a large fractionalisation of the votes lead to over-representation of large parties, but to an over-representation of small parties in a system with a dominant party.

The paper proceeds as follows: firstly positive vote transfer systems are defined and the relevant literature is reviewed. The formal model is developed in the subsequent section. Section four introduces the Hungarian electoral system, and section five shows that the model closely fits with the electoral results of the 28 parties that entered the Hungarian parliament in the period 1990-2010.

2 Positive vote transfer systems
While previous work has mainly dealt with the positive vote transfer system as an empirical case, this paper aims at investigating the same systems from a theoretical perspective. This distinction is mainly due to Hungary, where the positive vote transfer system is an element of a more complex system. This section defines positive vote transfer systems, and discusses their potential strengths.
Positive vote transfer systems have been studied in the context of their application in Hungary (Benoit and Schiemann 2001, Hibbing and Patterson 1992, Schiemann 2001). The cross-national literature struggles to decide whether the Hungarian exceptionalism should be classified as a proportional or majoritarian electoral system. If the Hungarian system is classified as mixed-member proportional, then it is due to the positive vote transfer, the only compensatory element in the Hungarian electoral rules. Some consider it to be an intermediate case, belonging to the mixed-member majoritarian (MMM) family (Ferrara et al. 2005: 55, Thames 2005: 288), as a system comparable to mixed-member proportional systems, with only a partial compensatory effect (Hibbing and Patterson 1992), or as a "supermixed" type, which combines elements from mixed-member majoritarian and mixed-member proportional systems (Benoit 2005: 235, Massicotte and Blais 1999: 357).

Similar to MMP systems, positive vote transfer systems are based on two tiers. The first tier (usually) consists of single-seat districts, from which candidates are directly elected to parliament by plurality or majority rule. The second tier consists of compensation mandates for those parties which do badly in the single-seat districts. Positive vote transfer systems differ however in their compensation mechanism. In positive vote transfer systems, the overall proportionality is achieved by considering votes for non-elected district candidates in the seat allocation: All votes for candidates who are not elected in the district tier pass through to the second tier (compensation tier), and are pooled by party. Seats in this tier are allocated proportionally to the transfer votes to parties. At the end of the day, every vote has two opportunities to contribute to parliamentary representation: either because the candidate for whom the vote was expressed was elected, or because it contributed to a seat gain for the candidate's party in the compensation tier. By default, positive vote transfer systems consist of only one vote. Negative vote transfer systems rely on a very similar mechanism.iv Earlier studies have shown to what degree positive vote transfer systems have reduced disproportionalities from the district tier (e.g. Hibbing and Patterson 1992), but have not studied this question systematically.

The positive vote transfer mechanism is immune to a specific type of strategic voting. Common MMP rules offer a loophole, allowing political parties and their voters to split their votes, casting their district vote for one party, and their list vote for one of its political allies. This strategy disables the proportional compensation mechanism and allows large parties and their allies to win considerably more seats than according to the rules of proportionality. Essentially, this strategy converts the seat allocation under MMP systems into an MMM-like outcome, to the detriment of small parties (Elklit 2008, Bochsler 2010: 123-9). Positive vote transfer systems do not allow for such manipulation, as the allocation of compensation mandates solely relies on votes expressed for non-elected candidates. Hence, there is no possibility that an individual’s vote can count for the election of a candidate of one party, and for the party list of another party, without compensation.v
3 Theoretical model
This section highlights in which situations positive vote transfer systems are fully proportional, and defines the variables affecting the proportionality of these systems. The theoretical model shows that both institutional variables and strategic behaviour have an important impact on the proportionality of positive vote transfer systems.

Studies of common MMP systems have argued that their capacity to proportionalise the electoral outcome depends on the size of the compensation tier. Different estimations indicate that around 25-50% of the overall seats in parliament need to be allocated as compensatory seats. As positive vote transfer systems use a different compensation mechanism than MMP, these estimates cannot be applied to them. Most importantly, they need more compensation seats than MMP systems for proportional outcomes.

In a first step, we formalise the seat allocation of positive vote transfer systems in multi-party systems. This allows us to identify when the system works proportionally, i.e. when each vote is represented with the same weight in parliament. We illustrate the seat allocation process in table 1, for a multi-party system with parties, A, B, etc. These parties win \( v_A, v_B, \ldots \) votes (absolute numbers or national vote shares) at the national level. We do not need to make any assumptions about how many single-seat district mandates \( S_{A,SSD}, S_{B,SSD}, \ldots \) these parties win. Each of these seats is won with a certain number of votes, i.e. the votes expressed for the victorious candidate in this district, \( q \). In two-round elections, \( q \) refers to the number of votes in the first round. This variable will vary considerably between two-party situations, where seats are won with more than 50% of the votes, and multi-party competitions, with many smaller parties, where some 30-35% of the district votes can be sufficient to win a seat. We assume that on average, \( q \) is equal for all parties winning seats in districts. This assumption does not mean that parties are equally strong, as it only refers to those districts where they win seats (i.e. we would compare the regional strongholds of a small party with the average district for large parties). If this assumption is violated, a proportional seat allocation is very unlikely to result. This would be the case if there were strong partisan effects of gerrymandering, malapportionment, or unequal turnout, or if there were some particular forms of unequal distribution of party sizes across the territory. While minor differences, as they typically occur, will not have very substantial effects on the model, major instances will amplify the disproportionalities.

We can now re-count: party A wins \( v_A \) votes, \( S_{A,SSD} \) district seats. Its candidates elected from these districts gained \( q \cdot S_{A,SSD} \) votes. Hence, for the candidates which were not elected in their districts, \( v_A - q \cdot S_{A,SSD} \) votes were expressed. These votes will be transferred to the compensation tier, and shall be called transfer votes. The same calculation can be made for every other party, which passed the electoral threshold (if there is one).
### Table 1: Calculating positive vote transfers.

<table>
<thead>
<tr>
<th>Party</th>
<th>Votes</th>
<th>number of district mandates won</th>
<th>votes in won districts</th>
<th>transfer votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$v_A$</td>
<td>$S_{A,SSD}$</td>
<td>$q \cdot S_{A,SSD}$</td>
<td>$v_A - S_{A,SSD} \cdot q$</td>
</tr>
<tr>
<td>B</td>
<td>$v_B$</td>
<td>$S_{B,SSD}$</td>
<td>$q \cdot S_{B,SSD}$</td>
<td>$v_B - S_{B,SSD} \cdot q$</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$ (below threshold)</td>
<td>$W$</td>
<td>$r$</td>
<td>$q \cdot r$</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>$v_T$</td>
<td>$S_{SSD}$</td>
<td>$q \cdot S_{SSD}$</td>
<td>$(v_T - w) - (S_{SSD} - r) \cdot q$</td>
</tr>
</tbody>
</table>

If there is a legal national threshold, we denominate votes expressed for parties below the threshold (which do not take part in the proportional seat allocation) as ‘wasted votes’ $(w)$, even if these parties might win certain district seats $(r)$.

Compensation seats $(S_{\text{comp}})$ are allocated in proportion to the transfer votes. Based on the number of compensation seats $S_{\text{comp}}$, we can also establish the ratio $u$, which expresses the number of transfer votes needed to win a single compensation seat.\textsuperscript{xiii}

$$u = \frac{(v_T - w) - (S_{SSD} - r) \cdot q}{S_{\text{comp}}}$$  \hspace{1cm} (1)

In the second step we establish the conditions under which it delivers an overall proportional seat allocation.

The seat allocation is proportional if the rate of seats per votes $A_i = \frac{S_{i,SSD} + \frac{v_i - S_{i,SSD} \cdot q}{u}}{v_i}$ is equal for all parties $i$ above the threshold. Or, for the two-party situation, this reads as follows:

$$\frac{S_{A,SSD} + \frac{v_A - S_{A,SSD} \cdot q}{u}}{v_A} = \frac{S_{B,SSD} + \frac{v_B - S_{B,SSD} \cdot q}{u}}{v_B}$$  \hspace{1cm} (2)

There are two possible solutions. First, $\frac{S_{A,SSD}}{v_A} = \frac{S_{B,SSD}}{v_B}$, if every party is equally well represented in the single-seat districts, then any number of compensation seats would lead to a proportional outcome. As this is usually not the case (especially if there are more than two parties), the second, more universal solution might be more relevant, according to which $u=q$.

Hence, proportional seat allocation occurs if the ratio at which transfer votes are converted into compensation seats is equal to the average number of votes cast for victorious candidates in the single-seat districts $q$. 

\textsuperscript{xiii} The value $u$ represents the number of transfer votes needed to win a single compensation seat.
Based on formula 1, we can now calculate the number of compensation seats needed for full proportionality.

\[
\frac{S_{\text{comp}}}{S_T} = \frac{1 - \frac{q \cdot (S_{\text{SSD}} - r)}{v_T - w}}{1 + \frac{q \cdot r}{v_T - w}}
\]  

(3a)

In special situations, if all parties pass the threshold (w=0), and no non-partisan candidates win any district seats (r=0), the formula reads more parsimoniously as:

\[
\frac{S_{\text{comp}}}{S_T} = 1 - \frac{q \cdot S_{\text{SSD}}}{v_T}
\]  

(3b)

This is equal to one minus the average vote share at which district candidates win their seat. Thus, if district mandates are won, on average, with 40% of the district votes, then a proportional outcome would be possible if 60% of the parliamentary seats were allocated in the compensation tier. Large amounts of wasted votes (on parties below the national threshold) would decrease the size of compensatory tier, but if these parties win district mandates, the size of the compensation tier would increase again. Online Appendix A develops this argument further, showing the consequences of a lack of compensation mandates on the degree of proportionality.

We have further illustrated this calculation for an imaginary country with 100,000 voters, 100 equally large single-seat districts, and three political parties A, B, C (above the threshold), winning 95% of the votes. 5% of the votes are cast for parties which fail to enter parliament. Each of the three parties wins seats in the single-seat district tier, with an average vote share rate of q=500. The largest party A wins 40,000 votes, and 60% of the district seats. This means that overall, 30,000 votes were expressed for successful district candidates of party A, and further 10,000 votes in non-successful districts (transferred to the compensation tier). Party B wins 30,000 votes, and as 15,000 were expressed for 30 winning candidates, the 15,000 remaining votes are transferred to the compensation tiers. Finally, party C, wins 25,000 votes, wins 10 single-seat district mandates with 5000 votes, and transfers 20,000 votes. Other parties remain below the threshold. To convert the total of 45,000 votes in the compensation tier at the same rate q=500 as the average winning rate in the district tier, 90 compensation seats are needed.

**Table 2: Calculation example: Positive vote transfers.**

<table>
<thead>
<tr>
<th>Party</th>
<th>votes</th>
<th>number of district mandates won</th>
<th>votes in districts won</th>
<th>transfer votes</th>
<th>compensation seats</th>
<th>overall number of mandates for proportional seat allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40,000</td>
<td>60 (60%)</td>
<td>30,000</td>
<td>10,000</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>(40%)</td>
<td>30 (30%)</td>
<td>15,000</td>
<td>15,000</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>B</td>
<td>30,000</td>
<td>30</td>
<td>15,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(30%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>25,000</td>
<td>10</td>
<td>5000</td>
<td>20,000</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>(25%)</td>
<td>(10%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>below</td>
<td>5000 (5%)</td>
<td>0 (0%)</td>
<td>0</td>
<td>0 (votes below threshold not eligible)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>threshold</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100,000</td>
<td>100</td>
<td>50,000</td>
<td>45,000</td>
<td>90</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>(100%)</td>
<td>(100%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 Implications of the model
The previous section has established how many compensation seats are needed for a positive vote transfer system to be proportional. In a system with large parties, where on average district candidates win their seats with 50% of the district votes, approximately half of the seats in parliament will be needed as compensation seats to allow for a proportional allocation. However, with multi-party competition, candidates with less than 50% of the votes in the first round can win district seats, and the size of the compensation tier needs to be larger in order for the system to be proportional. Therefore, the proportional or non-proportional properties of a positive vote transfer system – even with the same number of seats in each tier – will depend heavily on the political context and the strategic behaviour of voters and parties, and the degree to which they concentrate on a few strong candidates in the districts.

If the share of (really existing) compensation seats $\text{comp}_{\text{pr}}$ does not correspond to the necessary amount ($\text{comp}_{\text{pr}} = S_{\text{comp}} / S_T$), there might be different consequences:
- If the number of compensation mandates is lower (hence: $u > q$), then the parties which proportionally win more district mandates will remain over-represented overall. Thus, systems with fewer compensation mandates than needed tend to over-represent large parties.
- If there are more compensation mandates than needed (hence: $q > u$), then parties which do not win any, or only few district mandates will gain a disproportionally large number of compensation seats. This means that if there are many compensation mandates, small parties will be over-represented in parliament.

From this, we can derive a number of testable hypotheses.
- **Hypothesis 1:** If the share of compensation seats remains lower than the required amount, then parties that are over-represented in the single-seat tiers will also be over-represented in the overall seat allocation.
- **Hypothesis 2:** If the share of compensation seats is too low, lower numbers of compensation seats are associated with higher disproportionalities.
Hypothesis 3: At a given number of compensation seats below \( \text{comp} \), the lower average vote share \( q \) at which first-tier mandates are won, will lead to a more disproportional seat allocation.

The model entails a paradox: the very same electoral system might at one point over-represent large parties (if the number of parties is large, and thus \( q \) is low), and in a different election tend to over-represent small parties (if the number of parties is small, and thus \( q \) is high).

5 **How much compensation is there in Hungary**

The application of the positive vote transfer system in Hungary is suited for an empirical application of the model, as the degree of candidate coordination, introduced as \( q \) in the previous section, has substantially altered during the course of the electoral history of Hungary. Furthermore, Hungary’s electoral system has been designed in such a manner that the size of the compensation tier has dropped, from 90 to 64. As the Hungarian electoral system is more complex than our ideal theoretical type of a positive vote transfer system, this section discusses the applicability of the model.

The electoral system of Hungary consists of three tiers, and gives each voter two votes. The candidate vote determines the seat allocation in tiers 1 and 3, whereas the party vote (mainly) relates to the allocation of seats in tier 2 (see figure 1). The first tier consists of 176 mandates elected in single-seat districts (SSD) by majority rule. Votes cast for non-elected candidates are pooled according to the positive vote transfer rules, and transferred to the third tier. The second (list proportional) tier consists of 152 seats in twenty districts, with a 5% national legal threshold (4% in 1990), based on the party list votes. The third tier consists of a minimum of 58 compensation seats, and relies in its essence on the positive vote transfer, based on the votes for non-elected candidates in the first tier, for parties above the legal threshold. We can thus describe the Hungarian electoral system as a positive vote transfer system (tier 1+3) with an additional vote for a parallel proportional tier (tier 2), which is not linked to the seat allocation in tier 1 and 3. And as the proportional tier does not affect the disproportionalities, it should not alter the expected effect of the positive vote transfer system on the over- or under-representation of parties.

Some remainder seats from the second tier \(^{xiv}\) are moved to the third tier, along with the remainder party votes. The number of these votes is, however, rather small, so we disregard them. With the reduction of the number of parties, the number of seats moved from the second tier has declined drastically however, from thirty-two in 1990, to six in the 2006 and 2010 elections (table 3).
From 1990 to 2010, voters have increasingly defected from small parties, and the party system has changed from a multi-party system to a two-party system with some minor parties. This has altered the variables of our model over the six elections from 1990 to 2010. As a consequence, the number of votes ‘wasted’ for parties below the threshold has sharply decreased, whereas the vote share of victorious district candidates \( q \) has increased from 28% to 54%. The number of district seats won by parties below the threshold \( r \) remains low, with the remarkable exception of the Hungarian Democratic Forum (MDF), winning 17 district seats in 1998. Finally, in line with our assumptions, the number of votes for candidates winning district seats \( q \) is roughly equal across parties. (See standard error of \( q \) in table 3, and online appendix B for further tests.)

Based on this information, we have calculated for each election a theoretical measure of how many compensatory seats would be needed (\( \text{comp}_T \)) in order to ensure full proportionality. It varies from 45% to 66% of the parliamentary seats (not considering the second tier). The actual number of compensatory mandates (\( \text{comp}_R \)) has always been much lower, never exceeding 90, or roughly a third of the first and second tier. Judging from the model presented in this paper, the Hungarian constitution-makers opted for a partially compensatory system, and not for a fully proportional MMP system.

### Table 3: Parameters of the positive vote transfer system in the Hungarian elections, 1990-2010.

<table>
<thead>
<tr>
<th>Year</th>
<th>( q )</th>
<th>S.E.(q)</th>
<th>( \text{w (in SSD tier)} )</th>
<th>( S_{\text{compR}} )</th>
<th>( \text{comp}_R )</th>
<th>( R )</th>
<th>( \text{comp}_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>28.0%</td>
<td>3.8%</td>
<td>22.1%</td>
<td>58 + 32</td>
<td>33.8%</td>
<td>7</td>
<td>65.8%</td>
</tr>
<tr>
<td>1994</td>
<td>31.3%</td>
<td>3.5%</td>
<td>15.1%</td>
<td>58 + 27</td>
<td>32.6%</td>
<td>2</td>
<td>63.6%</td>
</tr>
<tr>
<td>1998</td>
<td>30.5%</td>
<td>3.5%</td>
<td>19.4%</td>
<td>58 + 24</td>
<td>31.8%</td>
<td>18</td>
<td>66.4%</td>
</tr>
<tr>
<td>2002</td>
<td>46.0%</td>
<td>3.0%</td>
<td>12.1%</td>
<td>58 + 12</td>
<td>28.5%</td>
<td>0</td>
<td>47.7%</td>
</tr>
<tr>
<td>2006</td>
<td>47.8%</td>
<td>4.4%</td>
<td>4.2%</td>
<td>58 + 6</td>
<td>26.7%</td>
<td>1</td>
<td>50.4%</td>
</tr>
<tr>
<td>2010</td>
<td>53.8%</td>
<td>3.2%</td>
<td>3.4%</td>
<td>58 + 6</td>
<td>26.7%</td>
<td>1</td>
<td>44.6%</td>
</tr>
</tbody>
</table>

Note: \( q \) and its standard deviation are weighted by the number of seats won in the district tier.  
\[
\text{comp}_T = 1 - S_{\text{SSD}} + q / (\text{SSD} - w + r S_{\text{SSD}})
\]
6 Empirical tests of the model
Relying on the results of Hungarian elections, this section provides some empirical evidence for the impact of the positive vote transfer system on disproportionality. First, we investigate overall measures of disproportionality, measured by the least-square index (Gallagher 1991). Second, we investigate party-based measures of under- and over-representation, measured by seat-vote-ratios (A) (Taagepera and Laakso 1980).

Throughout the analysis, we compare two manifestations of disproportionality. Disproportionality before compensation addresses the seat allocation in the district tier (first tier), and disproportionality after compensation addresses the overall seat allocation, hence the cumulated number of seats a party wins in all three tiers of the Hungarian electoral system.

In two-vote-systems, disproportionality can be calculated based on either of the two votes. For the question under study, the candidate votes appear as the more relevant baseline category for the calculation of disproportionality, as the compensation mechanism relies on this vote. However, we have also replicated our analyses based on the party votes.xv

National and district electoral results were obtained from the University of Essex, the Hungarian electoral commission, and are published in the author's district-level election database (Bochsler 2010).xvi

Figure 2 displays disproportionality for the six Hungarian elections. The arrows show the change of disproportionality before (start of the arrow, higher levels) and after compensation (end of the arrows). Disproportionality before compensation, based on the district votes (left panel), ranges from high (0.12) up to a figure (0.4) which is extraordinarily high in international comparison. After compensation, disproportionality drops to a degree of 0.05 to 0.17, hence it is on average cut by more than half, and reduced to a figure which is still high, but less extraordinarily so. Arguably, this reduction was due to an institutional learning process, where the party system underwent a concentration, responding to the incentives of the electoral system. The figure shows the reduction of disproportionalities in relation to q (displayed on the X-axis). At higher values of q, fewer compensation mandates are needed for a substantial reduction of disproportionality. The right-hand panel displays the results based on the party vote. Remarkable differences occur in 1998, when two parties (Fidesz and MDF) ran in an alliance in the proportional tier.

Figure 2: The average vote share of winning district candidates q (X-axis) and reduction of disproportionality through the positive vote transfer system (arrows, Y-axis).
Moving to the level of single political parties, we have more variance, which allows for limited multivariate tests. While \( q \) and the number of compensation seats \( S_{\text{comp}} \) are constant for all parties in the same election, seat-vote-ratios (over- or under-representation) in the district tier vary between parties. Seat-vote ratios above 1 mean that parties are over-represented (in comparison to their vote share), whereas below 1 means that they are under-represented. The seat-vote ratio after compensation is our dependent variable, the seat-vote ratio before compensation is one of the explanatory variables. As the compensatory tier approaches the ideal size, where it allows a proportional seat allocation, we expect the seat-vote-ratios after compensation to converge slightly above 1 for all parties, regardless of the seat-vote ratio before compensation. Our cases are all political parties which passed the 5% electoral threshold (1990: 4%).

Our argument states that compensation reduces the over- and under-representation of political parties. The capacity to reduce these disproportionalities depends on the size of the compensation tier, and the number of seats needed for compensation. Hence, we estimate interaction effects between disproportionality before compensation \((A)\) with the size of the compensation tier \(S_{\text{comp}}\), and in interaction with \( q \) (the average vote share at which district seats are won). Higher measures of \( S_{\text{comp}} \) and \( q \) are expected to lower the differences in the seat-vote ratios between parties. The effect should be linear (see appendix A).

Tests are based on OLS regressions with robust standard errors, considering also that standard errors for the same elections will be clustered by election year \(^{xvii} \) (table 4). The results are visualised in figures 4 and 5: parties with a seat-vote ratio before compensation of over 1 are still over-represented (for the empirically observed levels of \( S_{\text{comp}} \) and \( q \)). The graph becomes less steep, however, for increasing \( q \) and \( S_{\text{comp}} \). This indicates that more compensation seats and a higher average vote share of winning candidates in the single-seat districts lead to more proportional outcomes. Re-
Results should not be over-interpreted, given the inclusion of two (!) interaction terms with only a small number of cases. Note, however, that the results closely follow the theoretical model, and stay robust even if we switch to an alternative operationalisation of the dependent variable (measuring disproportionalities based on the party vote).

**Table 4: OLS regression explaining the overall seat-vote-ratio (after compensation). Seat-vote-ratios for parties in Hungarian elections 1990-2010. Robust standard errors (cluster for election years).**

<table>
<thead>
<tr>
<th></th>
<th>Baseline: single-seat district votes</th>
<th>Baseline: PR votes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coeff.</td>
<td>r.s.e.</td>
</tr>
<tr>
<td>ASSD</td>
<td>4.13</td>
<td>0.99</td>
</tr>
<tr>
<td>ASSD * compR</td>
<td>-8.88</td>
<td>2.43</td>
</tr>
<tr>
<td>ASSD * q</td>
<td>-2.75</td>
<td>0.63</td>
</tr>
<tr>
<td>compR</td>
<td>11.22</td>
<td>2.98</td>
</tr>
<tr>
<td>Q</td>
<td>2.90</td>
<td>0.73</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.79</td>
<td>1.18</td>
</tr>
<tr>
<td>N</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>adj R²</td>
<td>0.969</td>
<td></td>
</tr>
</tbody>
</table>

Note: the second specification excludes parties which have coordinated their candidates in an alliance in the single-seat districts (Fidesz-SZDSZ in 1990, Fidesz-MDF in 1998).

**Figures 4 & 5: Translation of the seat-vote-ratio in the single-seat tier (before compensation, X-axis) on the overall seat-vote-ratio (after compensation, Y-axis), moderated by compR (left graph) and by q (right graph).**

Seat-vote-ratios for parties in Hungarian elections 1990-2010, based on the single-seat district votes (table 4, left side).

**7 Conclusions**

Positive vote transfer systems are a rare, and seldom investigated type of electoral system, which are often assumed to proportionalisce election results so that the seat distribution in parliament corresponds to the parties' vote shares. With only a few cases of empirical applications available, this paper tackles
the research question from a formal, theoretical perspective, inspired by Taagepera's (2008) formal models on electoral system effects. It shows that proportional outcomes under positive vote transfer systems tend to be the exception rather than the rule, and the very same electoral system can lead to very different outcomes (see also Farrell and Katz, as well as Bedock and Sauger in this issue).

The inter-party effect of the positive vote transfer system is mediated both by institutional variations and by strategic behaviour. On the one hand, the size of the compensation tier of positive vote transfer systems needs to be reasonably large in order to allow for a proportional allocation of seats. In some cases, two thirds of the parliamentary seats need to be allocated as compensation mandates. On the other hand, strategic behaviour alters the required size of the compensation tier, and therefore impacts on the capacity of positive vote transfer systems to allow for proportional seat allocation: With fewer candidates per district, and if voters concentrate their votes on those most likely to be successful, the size requirements of the compensation tier drop. In highly fragmented party systems, and if voters do not vote strategically, the number of compensation seats must be larger.

In all cases where the compensation tier is not the ideal size, surprises are possible. Yet the effects could appear paradoxical: if there are fewer compensation seats than needed, the positive vote transfer will over-represent large parties. With more compensation seats than needed, small parties will be over-represented. As the necessary size of the compensation tier varies with the strategic behaviour of voters and parties, the very same electoral system can over-represent large parties if there are many small parties, and over-represent small ones, if there are few parties in the system as a whole.

Acknowledgements
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The proportionality of the seat allocation under MMP systems is almost taken for granted, so that quite a few scholars also have addressed them as “personalised PR systems” (among others, Kaase 1984, Colomer 2004: 49, Kreuzer 2004).

In Italy it was applied as ‘scoporo totale’, in the Senate elections from 1993 to 2005. Others used terms such as “MMM [mixed-member majoritarian] systems with partial compensation” (Shugart and Wattenberg 2001b: 20), systems with negative/positive vote transfers (Ferrara and Herron 2005: 22) or “compensatory systems” (Nohlen 2006: 216).

In order to re-establish a proportional seat allocation, the size of parliament was increased from 334 to 412 seats.

As ‘scorporo parziale’, it was used for the election of the Italian Chamber of Deputies. Votes for non-successful candidates count in the compensation tier, but the party of the victorious candidate also receives some votes: it receives the vote difference between the victorious candidate and the runner-up. While in positive vote transfer systems, extra votes for strong candidates do not affect the result, in negative vote transfer systems they can help a party to win additional compensatory seats.

In systems with two rounds for the district elections, strategic manoeuvres between the two rounds might potentially undermine the compensation mechanism. This can occur in districts where the same alliance of parties enters with two candidates in the second round.

Estimations are based on empirical examples (Moser and Scheiner 2004: 580, Cox and Schoppa 2002: 1029-30), rigorous empirical studies (Behnke 2003, Behnke et al. 2003) or theoretical models (Bochsler 2007).

Under MMP systems, no compensation mandates will be allocated to a political party which won sufficient district seats in order to be represented proportionally in parliament. In contrast, the positive vote transfer also allocates mandates to parties which already hold as many seats as they are entitled to according to their proportional result. For instance, in the 1994 Hungarian elections, the Socialists (MSZP) won 31% of the votes in the single-seat districts, but 85% of the district seats. Still, they gained 7 out of 85 compensation mandates. In the end, the party obtained 54% of all seats.

If the seat allocation in the compensation tier occurs separately in each region, the model can be applied at the regional level.

For negative vote transfer systems, we can calculate using the number of votes expressed for the second-strongest candidate in the district plus 1, instead of the votes for the strongest candidate. This will lower \( q \) slightly.

The model would result in a system of \( n \) quadratic equations, one for each party, and will in most cases not have a solution.

There is no malapportionment if the number of registered voters in districts won by a specific political party does not differ greatly from the average number of registered voters across all districts. Countervailing tendencies of the different biases can cancel each other out: The number of votes is the product of the district size (number of registered voters, relating to malapportionment), turnout and the vote share at which seats are won.

This might be the case for the Hungarian electoral reform passed in 2011, which among others contains an element of gerrymandering (Szigetvári et al. 2011).

We disregard differences resulting from the rounding up or down processes involved in the seat allocation rules.

For the 28 parties above the 5% threshold, the vote shares in the two tiers correlate at .99, and the party-based disproportionality measures (seat-vote-ratios) at .90. In four cases, parties have coordinated their candidates in the single-seat districts so that the votes expressed in the district race do not represent the strength of the parties in the PR tier.

University of Essex: http://www.essex.ac.uk/elections/; http://www.valasztas.hu [last access: 22 June 2010]

We have also run the model with clusters for political parties. The robust standard errors for the first specification tend to decrease slightly, and to increase slightly for the second specification.